Four and six parameter models of forced random noise respiratory impedance in normals

H. Lorino, C. Mariette, A.M. Lorino, A. Harf

ABSTRACT: Total respiratory Impedance has been measured between 3–42 Hz by the forced random noise technique, in 15 subjects breathing either air or a helium oxygen mixture in three experimental conditions: at basal state, and then with a resistor or a tube added at the mouth. Impedance is modelled, either by a 4-parameter model (M4), derived from the series model (resistance, inertance, compliance) by making resistance a linear function of frequency; or by a 6-parameter model (M6) including a central compartment (airway resistance and gas inertance), and a tissue compartment (resistance, inertance and compliance in series) placed in parallel with alveolar gas compliance. The additive resistance is perfectly evaluated by both models, whereas the additive inertance is not accurately estimated by the model M6, the fitting of which combines the real and imaginary parts of impedance. Resistance extrapolated at zero frequency on the one hand, inertance of M4 and central inertance of M6 of the other, are highly correlated. However, changes in some parameters of both models according to the experimental conditions are difficult to explain on physiological grounds. We conclude that the model M6 cannot be easily and accurately identified over such a limited frequency range, at least in normals, while the model M4 yields a simplified description of Impedance which may be sufficient for diagnostic purposes.

When a forced excitation is applied to the mouth of a subject, the mechanical function of the respiratory system can be characterized by the frequency dependence of its input impedance \( Z = R + jI \), which expresses the relationship between mouth pressure and flow. Impedance data are often interpreted using linear lumped-parameter models made of resistive, inertial and compliant elements. The model M6, first proposed by DuBois et al. [1] and then used by other authors [2–5], includes an airway compartment with airway resistance \( R_1 \) and gas inertance \( I_1 \), a gas compartment with gas compressibility \( C_1 \) in parallel with a tissue compartment made of a resistance \( R_2 \), an inertance \( I_2 \) and a compliance \( C_2 \) arranged in series (fig.1a). A much simpler model, the model M3, made of a resistance \( R \), an inertance \( I \) and a compliance \( C \) arranged in series (fig.1b), has often been used to fit impedance data in normals [6–12]. However, this model fails to represent any frequency dependence of the real part of \( Z \) (ZR), or any change of this dependence according to the gas mixture breathed. That is why it was sometimes substituted to constant \( R \) a linear function of frequency \( f \) [13, 14]: \( ZR = R + S \cdot f \), which produced a new model M4 (fig.1c).

There is very little information in the literature concerning a detailed comparison of a series model like M4 and the model M6 in the same subjects. The purpose of this study was, thus, to compare the ability of both models to detect an additive resistance or inertance, and to analyse the relationship between the homologous parameters of M4 and M6, in fifteen normal subjects breathing air or a (22% oxygen (O_2), 78% helium (He) mixture.
MODELS OF RESPIRATORY IMPEDANCE

Methods

We used the forced random noise technique previously described [6, 13], and the experimental set-up schematized in fig. 2. Two loudspeakers were excited by a random signal in the 3-42 Hz bandwidth (Oscillomat, EMA, ZI Plaisir, France). Mouth flow was measured with a screen pneumotachograph (Jaeger Lily, resistance=0.35 cmH2O·l·s) connected to a differential pressure transducer (Sensym LX 06001D). The common mode rejection ratio of this flow measuring system was about 50 dB at 40 Hz. Mouth pressure was measured with an identical pressure transducer referenced to the atmosphere. The two transducers were matched within 1% of amplitude and 2° of phase. The pneumotachograph and the generator were flushed by a constant bias flow (about 0.3 l·s) of compressed air or (22% O2, 78% He) mixture. The gain factors were obtained using a slanted fluid manometer for pressure and a 1 litre syringe for flow. With the He-O2 mixture, the gain of the pneumotachograph was found to increase by 12%. The subject was seated, wore a noseclip and was asked to support his cheeks firmly and to breathe quietly. The measurements with He-O2 was started after a wash-in period of at least one minute.

Fig. 2. – Diagram of the apparatus; M: mouthpiece; Pnt: pneumotachograph; Pt: differential pressure transducer; Lb: loudspeaker box; Pa: power amplifier; Frn: forced random noise generator; Lpf: low-pass filter; ADC: analogue-to-digital convertor.

Fifteen normal adult subjects (7 males, 8 females) were studied in three experimental conditions:

E1: the subject alone (control condition);
E2: with an additive piece of tubing (51 cm in length, 2.4 cm ID); according to its physical nature, such a device should be represented by a spatially distributed model, which would take into account the shunt impedance due to gas compressibility. The impedance of the tube was, therefore, analysed in the following ways;

a) without any load, the tube behaved as a pure inerance (about 0.015 cmH2O·l·s in air and 0.006 in He-O2, negligible resistance);
b) when an external load equivalent to a normal subject was added (mechanical model made of a resistance, an inerance and a compliance), the tube by itself behaved as the same lumped inerance as in the previous case, whilst its resistance gradually departed from zero above 15 Hz, to reach about 0.5 cmH2O·l·s at 30 Hz and 1 cmH2O·l·s at 42 Hz in air, this frequency dependence being much less pronounced in He-O2. When the total impedance of the set-up was measured, inerance was found to be the sum of the lumped inerances of the tube and the physical model, and resistance was roughly the sum of the lumped resistance of the physical model and the frequency dependent resistance of the additive tube. It was, therefore, assumed in this study that the tube could be considered as an additional lumped inerance in the 3-42 Hz frequency range;

E3: and with an additive resistor (micromesh wire screen mounted in a plastic ring and inserted in a short piece of tubing). The experiments described above were applied to the resistor, and proved that its resistance was constant over the 3-42 Hz range (about 2.3 cmH2O·l·s in air and 2.5 in He-O2). Inertance of this device was about 0.005 cmH2O·l·s in air and 0.002 in He-O2.

In each condition, four 12 s records were lowpass filtered to prevent aliasing (8th order Butterworth, cutoff frequency = 42 Hz) and digitized at a frequency of 128 Hz. Spectral analysis was applied to contiguous 4 s periods using a 512 points FFT algorithm and a 0.25 Hz frequency resolution. The resulting twelve spectra were averaged, using a 0.5 Hz frequency step, to yield one estimate of impedance data in each condition, firstly for air, and then for helium breathing. Coherence between pressure and flow was also calculated as a function of frequency [15]: only those impedance data corresponding to a value of the coherence function higher than 0.9 were used to calculate parameters of both models from the average set of impedance data. Using this coherence threshold, the average percentage of impedance data submitted to models identification was 76% in (E1, Air) and 78% in (E2, Air), with a lower bound of frequency between 3–6 Hz (in three subjects, only few impedance measurements were available below 10 Hz), whereas it was higher than 95% in the other conditions, with a good representation of the low frequencies.

Fig. 3. – Example of impedance data fitted by models M4 (a) and M6 (b), in condition (E1, Air). ZR is the real part, and ZI the imaginary part, of impedance, represented as functions of the frequency f. Units are Hz and cmH2O·l·s.

Values of the parameters of the model M4 were obtained by applying linear regression analysis separately to the real (ZR) and imaginary (ZI) parts of impedance: ZR=R+S·f, and ZI=Io·C·f, where o = 2π f. Estimates
of the parameters of the model M6 were yielded by a non-linear regression technique, the Gauss-Marquardt method [16], an iterative algorithm which progressively minimized the quadratic distance between the measured and the theoretical complex impedances (Appendix 1). An example of fitting of models M4 and M6 to experimental impedance data is given in fig. 3.

Statistical analysis of the parameters of the models used the paired t-test, linear regression and correlation (p<0.05 was considered significant).

In general, the quality of the coherence was improved in all individuals in two experimental circumstances: by addition of the resistor (comparison of E1 and E3), especially at the low frequencies and for air breathing: by substitution of He-O\textsubscript{2} to air in each of the three conditions.

The mean parameter estimates for the M4 model are given in table 1, together with similar results obtained by

| Table 1. - Values of the parameters obtained with M3 and M4 models |
|-------------------|-----------------|---------|--------|
|                   | R               | S       | C      | f\textsubscript{max} | n   |
| MICHAELSON et al. [6] | 2.08 (20) | 14.6 (20) | 29.3 (35) | 40 | 10 |
| HAYES et al. [7] a) | 2.09 (21) | 7.4 (32) | 53.1 (58) | 30 | 12 |
|                    | 2.42 (20) | 6.5 (29) | 41.5 (43) | 30 | 15 |
| PESLIN et al. [8]  | 2.62 (30) | 10.7 (30) | 26.9 (30) | 20 | 43 |
| FIMMEL et al. [9]  | 2.66 (24) | 6.8 (38) | 57.1 (24) | 35 | 5 |
| FIMMEL et al. [10] | 2.24 (21) | 12.3 (17) | 51.0 (14) | 25 | 5 |
| PESLIN et al. [11] | 2.11 (18) | 11.9 (13) | 38.0 (26) | 30 | 10 |
| ROTGER et al. [12] | Air 1.82 (22) | 9.7 (35) | 28.6 (24) | 30 | 10 |

this study

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>I1</th>
<th>I2</th>
<th>C1</th>
<th>C2</th>
<th>f\textsubscript{max}</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>PESLIN et al. [4]</td>
<td>1.24 (45)</td>
<td>1.05 (83)</td>
<td>14.5 (23)</td>
<td>1.4 (100)</td>
<td>after</td>
<td>31.8 (35)</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

this study

|                    | E1, Air 2.18 (26) | 0.5 (40) | 13.9 (16) | 3.6 (33) | 8.8 (45) | 20.3 (30) | 42 | 15 |
|                    | E1, He 1.57 (29) | 0.81 (55) | 6.9 (17) | 1.2 (80) | 7.6 (46) | 23.1 (35) |
|                    | E2, Air 1.71 (31) | 1.1 (31) | 24.6 (12) | 6.9 (28) | 9.1 (62) | 30.1 (37) |
|                    | E2, He 1.83 (23) | 0.37 (97) | 11.8 (14) | 2.4 (44) | 9.8 (53) | 18.3 (30) |
|                    | E3, Air 4.42 (9) | 0.47 (33) | 18.0 (14) | 3.9 (30) | 8.8 (52) | 24.2 (29) |
|                    | E3, He 4.1 (8) | 0.74 (54) | 9.5 (13) | 2.2 (50) | 8.7 (25) | 21.5 (27) |

R1: airway resistance; I1: gas inertance; C1: gas compliance; R2, I2, C2: resistance, inertance and compliance of tissues. Values are mean (percentage coefficient of variation) ; TGV: thoracic gas volume; f\textsubscript{max}: upper limit of the frequency range, in Hz; n: number of subjects. Units are cmH\textsubscript{2}O·l\textsuperscript{-1}·s\textsuperscript{-1} for R, 10\textsuperscript{3}·cmH\textsubscript{2}O·l\textsuperscript{-1}·s\textsuperscript{2} for I and 10\textsuperscript{3}·l·cmH\textsubscript{2}O·l\textsuperscript{-1} for C.
MODELS OF RESPIRATORY IMPEDANCE

previous authors for model M3 in control condition E1. By comparison with air, the He-01 mixture slightly reduced R in condition E1, but not in condition E3. The values of R in condition E2 were significantly lower than in the control condition, especially for air breathing. However, when linear regression was restricted to the 3-25 Hz range, the mean value of R became 2.3 cm H2O L-1 s in air and 1.95 cm H2O L-1 s in He-O2. The frequency dependence of ZR, as measured by S, significantly decreased from air to helium in the three conditions. If condition E1 is taken as the reference, addition of the resistor (E3) did not significantly modify the value of S, whereas addition of the tube (E2) significantly increased S, whatever the gas mixture. The helium/air ratio of average inerance I is 0.47, 0.46 and 0.51 in the condition E1, E2 and E3, respectively. Compliance (C) significantly decreased from air to helium in three conditions, but it was not significantly modified by addition.
of either the resistor or the tube, whatever the gas mixture.

The mean estimates of the parameters of the M6 model are listed in table 2, and final uncertainty of each parameter ($U(p)$ in Appendix 2), is pictured in figure 4. The lowest coefficients of variation and uncertainties were obtained for inertance $I_1$ and resistance $R_1$, whereas resistance $R_2$ and compliance $C_1$ generally appeared as the most variable parameters among individuals and the most uncertain parameters. By comparison with control condition $E_1$, mean uncertainty was lowered for all the six parameters by addition of the mechanical resistor, especially for He-O$_2$ breathing, whereas addition of the tube considerably increased uncertainty of the resistance $R_1$. Compliance $C_1$ was about twice the value which can be derived from thoracic gas volume, except in condition $(E_2, \text{Air})$ where $C_1$ was lower than this estimate. Resistance $R_1$ significantly decreased from control condition to condition $E_2$ in air, as well as from air to helium in conditions $E_1$ and $E_3$. The helium/air ratio of average inertance $I_1$ was 0.50, 0.48 and 0.53 in the conditions $E_1$, $E_2$ and $E_3$, respectively. The $R_2$ and $I_2$ parameters were significantly modified from air to helium in the three conditions, whereas $C_2$ was modified in condition $E_2$ only.

The mean differences between conditions $E_2$ and $E_1$ for inertance, $E_3$ and $E_1$ for resistance, are given in table 3: additive resistance is accurately detected by both models, but additive inertance is imperfectly evaluated by central inertance $I_1$ in the model M6.

Several comparisons have been drawn between the correlative parameters of the two models. According to table 4, the resistances $R$ (model M4) and $RT$ (model M6) extrapolated at zero frequency were strongly correlated, except in condition $(E_2, \text{Air})$. The regression coefficients were reasonably close to unity, except in condition $(E_3, \text{Air})$ where the constant term approached $1 \text{cmH}_2\text{O} \cdot \text{L}^{-1} \cdot \text{s}^{-2}$. Table 5 indicates that the coefficient of the regression between inertance $I$ in M4 and inertance $I_1$ in M6 departed slightly from unity only in the conditions $(E_2, \text{Air})$, with a high constant term, and $(E_3, \text{He})$.

Table 3. – Estimation of additive elements by the M4 and M6 models ($\Delta R$ between conditions E3 and E1, $\Delta I$ between conditions E2 and E1)

<table>
<thead>
<tr>
<th>Model</th>
<th>Air</th>
<th>He</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta R$</td>
<td>$\Delta I$</td>
<td>$\Delta R$</td>
</tr>
<tr>
<td>M4</td>
<td>2.2 (16)</td>
<td>14.5 (14)</td>
</tr>
<tr>
<td>M6</td>
<td>2.2 (18)</td>
<td>-0.03 (25)</td>
</tr>
</tbody>
</table>

$R$ and $I$: resistance and inertance from model M4; $R_1$, $R_2$, $I_1$ and $I_2$: central and peripheral resistances and inertances from model M6. Values are mean (percentual coefficient of variation). Units are $\text{cmH}_2\text{O} \cdot \text{L}^{-1} \cdot \text{s}^{-2}$ for resistances and $10^3 \text{cmH}_2\text{O} \cdot \text{L}^{-1} \cdot \text{s}^{-2}$ for inertances.

Discussion

To date, very few results obtained by applying model M6 to respiratory input impedance have been fully documented in the literature [4] and, to our knowledge, the M4 and M6 models have never been compared in the same group of normals. Identification of model M4 is obviously straightforward and provides a unique value for each parameter, whereas identification of model M6 proves to be more hazardous: the final optimal values of the parameters have been found to be somewhat dependent on the initial values adopted to start the identification process, so that they are not likely to be unique. A similar conclusion was formulated by Pau1in et al. [4] when using a simple Gauss algorithm. This is probably due to the fact that, over the limited frequency range adopted in our study, the frequency dependence of $Z_R$ is not marked, except in the condition $(E_2, \text{Air})$ where it is probably mainly due to the mechanical behaviour of the tube. DorKIN et al. [17] reported that marked extrema of $Z_R$ and $Z_I$, which would allow more accurate calculation of the parameters of M6, might not be observable below $100 \text{Hz}$ in normals. However, they showed that impedance data obtained up to $200 \text{Hz}$ could no longer be correctly fitted by the model M6 which had to be
MODELS OF RESPIRATORY IMPEDANCE

transformed in a seven parameters model M7, with R1 becoming a linear function of frequency [17].

As a preliminary remark about experimental impedance data, it has been observed, in all individuals and in the three conditions, that breathing He-O₂ tends to lessen and make more linear the frequency dependence of ZR, and to increase the resonant frequency (i.e. the frequency at which ZI is zero). However, it has been proved [11, 12] that upper airway wall motion may be responsible for errors in the estimation of the frequency dependence of impedance by the conventional method, i.e. when the forced oscillations are applied at the mouth, because this method is sensitive to the shunt impedance of upper airways even when the cheeks are firmly supported [11]. By using a head generator method as the reference, together with a model similar to M4, Rønæs et al. [12] have demonstrated that the decrease in the frequency dependence of ZR is underestimated by the conventional method used in the present study. However, both methods evidence a significant decrease in this frequency dependence, which can result from the fact that the He-O₂ mixture reduces the contribution to total impedance of central airways, the resistance of which becomes density dependent in the presence of a flow regime which is not fully laminar [18]: in this case, the frequency dependence of resistance can be considered as inversely related to the kinematic viscosity which is about three times higher for He-O₂ than for air [19]. Rønæs et al. [12] also found that the conventional method significantly underestimates inertance: however, our estimates of inertance (I) in condition E1 (table 1) are substantially higher than those reported in their paper. Finally, their estimates of compliance (C) obtained with the conventional method were lower than ours, but with a helium/air ratio of 0.85 which closely compares with our value of 0.8 in condition E1 (table 1). In our study, the unexpected changes in the parameters C, R2, I2 and C2 observed when air is replaced by He-O₂ can be explained either by a failure of the proposed models to adequately describe the mechanical behaviour of the respiratory system, especially when the upper airway artefact is not corrected for [12], or by an imperfect identification of M4 and M6 over the 3–42 Hz bandwidth.

As illustrated in fig. 4, uncertainty evaluated for the optimal values of the M6 parameters depends on the type of parameter and the experimental condition envisaged. The parameters I1 and R1 were by far the best determined, probably because they become progressively preponderant in ZI and ZR with increasing frequency (Appendix 1). On the contrary, R2 and C1 appeared as the most uncertain parameters. We observed that the value of the resistance at zero frequency (RT in Appendix 1) was most often close to the value of ZR at low frequencies. Since RT was thus correctly identified and the standard error of R1 was low, except in condition (E2, Air), uncertainty about R2 was in direct relationship with that about C1 and C2. These compliance parameters are mainly determined at low frequencies, at which coherence is often spoiled by spontaneous breathing interferences, which probably explains why their uncertainty is high and why estimation of R2 can become dubious.

Compared to control condition E1, introduction of the additive resistor in E3 tended to reduce uncertainty of every parameter, and it has been found to markedly improve coherence at low frequencies, especially from (E1, Air) to (E3, Air). This improvement might be due to the fact that flow tended to be lower with the additional resistance, thus reducing either the influence of a possible non-laminar flow or the spontaneous breathing artefact generated by the subject. On the contrary, few acceptable results were obtained in condition (E2, Air), because the real part ZR of Z most often exhibited a pronounced frequency dependence which was non-linear in many subjects and, therefore, could not be adequately described by either model. This type of frequency dependence is probably attributable to the mechanical behaviour of the tube, in which gas compression induces such a non-linear frequency dependence of resistance: a shorter additive tube would probably have yielded more easily interpreted results.

The additive resistance, which is expected to add only a constant value to ZR over the frequency range used in this study, was accurately detected by both models (table 3). In the model M6, the mean change in the parameter R2 was low and not significant. In both air and He-O₂, the parameters I1 and I2 were increased by the same small amount which corresponded to a connecting piece of tubing. The comparison of the resistance parameters in the two models (table 4) shows that the zero frequency values, RT and R, were tightly correlated, with a regression slope generally close to unity and nearly constant in helium throughout the three conditions. The sum RS of the M6 resistances and R were also tightly correlated, but the regression slope clearly departed from unity in two conditions (E1, He) and (E3, He), which shows the contribution of the M6 compliances to the total resistance estimate RT. It must be noted that mean R1 was fairly close to mean R, except in these two conditions (tables 1 and 2). In conditions E1 and E3, increase in resistance R2 when breathing He-O₂ probably resulted from an incorrect separation of R1 and R2, since R1 simultaneously decreased in a similar proportion. Both the resistances R and R1 significantly decreased from (E1, Air) to (E2, Air). However, it is clear that, because of the non-linear frequency dependence of ZR induced by the tube, the regression line fitted to ZR by the M4 model yielded a quite unrealistic estimate of R in most subjects: indeed, if the regression was restricted to the 3–25 Hz frequency range, the average values of R become similar in conditions E1 and E2 for the two gas mixtures. One may assume that, like R, the R1 parameter was underestimated in condition (E2, Air), which explains the associated increase in the R2 parameter. As regards the condition E3, the fact that no change was observed for R when replacing air by He-O₂ may be explained by the combined effect of a decrease in respiratory resistance, as it was observed in condition E1, and an increase in additional resistance (see Methods).

When considering the inertial parameters, it clearly appears that inertance of the additional tube was better detected by M4 than by M6 which might have included a fraction of the additional inertance in I2, as indicated...
by the significant change in the I2 parameter and the fact that (\(\Delta I_1 + \Delta I_2\)) was close to \(\Delta I\) (table 3). This distribution of the additive invariance may appear somewhat surprising, since invariance \(I_1\) is the M6 parameter which exhibited the lowest uncertainty, and I2 was assumed to represent tissues invariance. However, since the quadratic criterion to be minimized combines the real and imaginary components of impedance, the frequency dependence of the tube resistance may have hindered correct identification of the I1 parameter. Analysis of the relationship between I and I1 (table 5) indicates that the regression slope was markedly different from unity in condition (E2, Air), for which correlation was barely significant, and in condition (E3, He), where no satisfactory explanation can be proposed. Examination of tables 1 and 2 confirms that mean airway invariance \(I_1\) was close to mean total invariance \(I_1\) in every condition, which corresponds to the fact that invariance is located for its major part in the airways [4]. As stated by Rotter et al. [12], the helium/air ratio of invariance \(I_1\) is expected to be equal to \((1+ab)/(1+b)\), where a is the gas density ratio (0.36 for 22% O2, 78% He), and b the ratio of airway and tissue invariance in air. Our values of the helium/air ratio of invariance \(I_1\) would, therefore, correspond to a contribution of airways to total invariance of 75–80%, whereas a contribution of about 90%, which would give an invariance ratio of 0.42, has been previously reported [4, 12]. The same contribution, evaluated from model M6 as the mean ratio \(I_1/(I_1 + I_2)\) throughout the six experimental conditions, is 81%. However, two surprising findings tend to demonstrate that the values of the invariance parameters in M6 do not have a clear physiological meaning: on the one hand, a substantial change of the so-called “tissue invariance” I2 observed from air to He-O2; on the other hand, the change in airway invariance (I1), which would be expected to be approximately in proportion with gas density, is only comparable to the change in total invariance (I).

As regards the compliance estimates, Appendix 1 indicates that the lower frequencies mainly determine the sum \((C_1 + C_2)\) in the model M6. It is, therefore, not surprising to observe that it was the sum \((C_1 + C_2)\), rather than either of these compliances, which remained reasonably constant in all the experimental conditions with model M6 (table 2). This sum was close to the value of C for He-O2 breathing (table 1), but significantly higher values of C have been obtained for air breathing. As the resonant frequency was always lower for air breathing, and compliance was mainly determined by negative reactance data, it is likely that fitting of this reactance by M4 is more uncertain for air breathing, so that C may be overestimated. Since the product of the I and C estimates was determined by the measured resonant frequency, invariance (I) would then be underestimated, which suggests that the helium/air invariance ratio may have been slightly overestimated by the model M4. The observed balance in the values of I and C might also be induced by the upper airway artefact [11] which can be largely reduced by applying forced oscillations around the head [12, 14]. Finally, it must be emphasized that the various estimates of C1 in model M6 clearly overes-

```
Z_m = R_1 + j\omega_1 + Z_r',
```

with \(\omega_1 = 2\pi f\), f=frequency, \(j^2 = -1\)

with \(1/Z_r' = j\omega_1 + (R_2 + j\omega_2 + 1/(jC_2))/R_2\).

Combination of these two equations yields the value of \(Z_{R_m}\) and \(Z_{I_m}\)

```
Z_{R_m} = R_1 + R_2 C_2/\omega D
Z_{I_m} = I_1\omega + N/(\omega D)
```

```
 MODELS OF RESPIRATORY IMPEDANCE

D=\((I2 \ C1 \ C2)^2 \omega^4 + ([R2 \ C1 \ C2]^2 - I2 \ C1 \ C2 \ (C1 + C2)) \omega^2 + (C1 + C2)^2\)
N=\(-I2^2 \ C1 \ C2^4 + (2 \ I2 \ C1 \ C2 + I2 \ C2^2 - R^2 \ C1 \ C2)^2 \omega^2 - (C1 + C2)^2\)

Similar expressions of \(Z_m\) as a complex function of \(s=j\omega\) are given in [21].

The value of \(ZR\) at zero frequency can be calculated as:

\[RT=ZR_m(0)=R1 + R2 \ (C2/(C1 + C2))^2\]

The real component \(ZR\) exhibits an extremum if it exists \(\omega_e=2\pi f_e\), such as \(d(D/d\omega)\) \(\omega_e=0\), that is to say if the following quantity is positive:

\[\omega_e^2=(1/I2) \ (1/C1 + 1/C2) - (R2/I2)^2/2\]

Using mean values found in reference [4], this quantity is negative. For mean values found in this study in condition (E1, A1), the frequency \(f_e\) is about 30 Hz.

As \(\omega\) increases, \(Z_R\) tends to be equivalent to \(R1\), and \(Z1\) to \((1-h/1/C1)0\), \(i.e\), the influence of the peripheral compartment tends to die away. As \(\omega\) decreases to zero, \(N/(\omega D)\) tends to be equivalent to \(-1/(\omega(C1 + C2))\), \(i.e\). the sum of the static elements placed in parallel.

The quadratic criterion to be minimized by the Gauss-Marquardt algorithm is:

\[\Delta^2 = \sum_i (ZR - Z_{R_m})^2 + (ZI - Z_{I_m})^2\]

where the summation takes into account only the \(n\) frequencies corresponding to a coherence higher than 0.9.

Appendix 2

Adopting a first order Taylor expansion of the model \(Z_m\) yields the residual variance of the M6 parameters [21]:

\[\text{var}(p_i)=s^2h_{ii}\]

where \(s=\Delta^2\) and \(\delta=(0-6)\) is the estimate of the variance of the measurement noise, \(\Delta^2\) is the minimal value of \(\Delta^2\), \(n\) is the number of frequencies used to calculate \(\Delta^2\), \(h_{ii}\) is the \(i^{th}\) element of the main diagonal of \(H^1\),

\[H=\sum_i SSS^t\]

with \(S=\frac{\partial Z_m(t)}{\partial p_i}\)

\(S\) is the vector of the sensitivity coefficients of the model \(Z_m\) calculated for the optimal values of the 6 parameters \(p_i\). Uncertainty of each parameter \(p_i\) is finally expressed as the percentual ratio of the residual standard error of each parameter to the optimal value of the parameter:

\[U(p_i)=100 \sqrt{(\text{var}(p_i)/p_i)}\]

Acknowledgements: The authors are grateful to B. Louis for helpful advice concerning fluid mechanics.

References


RÉSUMÉ: L'impédance respiratoire totale est mesurée entre 3 et 42 Hz par la technique du bruit aléatoire forcé à la bouche sur 15 sujets sains respirant de l'air ou un mélange He-0\textsubscript{2}, dans trois conditions: à l'état de base, puis après addition à la bouche d'une résistance ou d'un tube. L'impédance est représentée, soit par un modèle à 4 paramètres (M4), dérivé du modèle série (résistance, inerance, compliance) en rendant la résistance fonction linéaire de la fréquence, soit par un modèle à 6 paramètres (M6), comportant: un compartiment central (résistance de voies aériennes, inertance du gaz), et un compartiment tissulaire (résistance, inertance et compliance en série) placé en parallèle avec la compliance du gaz alvéolaire. La résistance additionnelle est parfaitement évaluée par les deux modèles, alors que l'inertance additionnelle est mal estimée par le modèle M6, dont l'identification combine les parties réelle et imaginaire de l'impédance. Les résistances extrapoltées à la fréquence nulle sont fortement corrélées, de même que l'inertance de M4 et l'inertance centrale de M6. Cependant, certains paramètres de l'un et l'autre modèles varient avec les conditions expérimentales de manière malaisément explicable au plan physiologique. Nous en concluons que, si le modèle M4 peut fournir une description simplifiée, à finalité diagnostique, de l'impédance jusqu'à 40 Hz, le modèle M6 ne peut probablement pas être identifié de manière aisée et sûre sur une plage de fréquence aussi réduite, du moins chez des sujets normaux. *Eur Respir J.*, 1989, 2, 874–882